Statstrive ExamIQ

ISI MSQE PEA 2022 Question Paper

Regenerated from the local solution TeX source. Items missing full source detail are marked Needs Review.

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Question 1

Topic: Consumer Theory **Difficulty:** Easy **Status:** Ready

Consider an economy with two goods X and Y. Let the utility function be given by $u(x, y) = A\sqrt{xy}$ where $A > 0$, $x \geq 0$ and $y \geq 0$. The budget constraint is $P_X x + P_Y y \leq M$, with $M > 0$. Let $P_X = P_Y > 1$ and let (x^*, y^*) denote the utility maximizing bundle. Then,

(A) it must always be that $x^* > y^*$

(B) it must always be that $x^* = y^*$

(C) it must always be that $x^* < y^*$

(D) it must always be that $x^* + y^* = M$

Question 2

Topic: Consumer Theory **Difficulty:** Easy **Status:** Ready

For the linear utility $u(x_1, x_2) = 3x_1 + 2x_2$ with budget $2x_1 + 3x_2 \leq M$, $x_1, x_2 \geq 0$, the Lagrangian is

$$L(x_1, x_2; \lambda) = 3x_1 + 2x_2 + \lambda[M - 2x_1 - 3x_2].$$

Determine the equilibrium $(x_1^*, x_2^*, \lambda^*)$.

(A) $(x_1^* = M/2,\ x_2^* = 0,\ \lambda^* = 3/2)$

(B) $(x_1^* = 0,\ x_2^* = M/3,\ \lambda^* = 2/3)$

(C) $(x_1^* = M/2,\ x_2^* = 0,\ \lambda^* = 2/3)$

(D) $(x_1^* = 0,\ x_2^* = M/3,\ \lambda^* = 3/2)$

Question 3

Topic: Consumer Theory **Difficulty:** Moderate **Status:** Ready

In month 1: $p_X = 2, p_Y = 3$. Consumer A chose $(3, 8)$, consumer B chose $(6, 6)$. In month 2: $p_X = 3, p_Y = 2$. Consumer A chose $(8, 3)$, consumer B chose $(4, 9)$. Which statement is correct?

(A) Both consumers satisfy WARP

(B) Neither consumer satisfies WARP

(C) Consumer A satisfies WARP but not consumer B

(D) Consumer B satisfies WARP but not consumer A

Question 4

Topic: Production and Cost **Difficulty:** Easy **Status:** Ready

Production function: $Y(L,K) = \min\{2L, K\}$; cost $C = wL + rK$, $w, r > 0$. Find (L^*, K^*) minimizing cost subject to $Y(L,K) \geq \bar{Y}$.

(A) $L^* = \bar{Y}$ and $K^* = \bar{Y}/2$

(B) $L^* = \bar{Y}$ and $K^* = \bar{Y}$

(C) $L^* = \bar{Y}/2$ and $K^* = \bar{Y}$

(D) None of the other options is correct

Question 5

Topic: Market Structure **Difficulty:** Moderate **Status:** Ready

Inverse demand $P = 1 - q_1 - q_2$; cost $c_i(q_i) = \kappa_i q_i$, $\kappa_i \in (0, 1)$. Find firm 2's Cournot equilibrium profit.

(A) $(1 - \kappa_1 + \kappa_2)^2/9$

(B) $(1 - \kappa_2 + \kappa_1)^2/9$ [i.e. $(1 - 2\kappa_2 + \kappa_1)^2/9$ corrected below]

(C) $(1 - 2\kappa_1 + \kappa_2)^2/9$

(D) $(1 - 2\kappa_2 + \kappa_1)^2/9$

Question 6

Topic: Consumer Theory **Difficulty:** Moderate **Status:** Ready

Which statement is correct in a two-good world?

(A) Diminishing MU of both goods is sufficient for diminishing MRS

(B) Diminishing MU of both goods is necessary for diminishing MRS

(C) Diminishing MU of at least one good is necessary for diminishing MRS

(D) Diminishing MU of at least one good is neither necessary nor sufficient for diminishing MRS

Question 7

Topic: Consumer Theory **Difficulty:** Easy **Status:** Ready

A non-transitive preference relation can be represented by a utility function

(A) Always

(B) Only if preferences are complete

(C) Only if preferences are complete and convex

(D) Never

Question 8

Topic: Consumer Theory **Difficulty:** Moderate **Status:** Ready

$U(x, y) = \min\{x, y\}$, income $M = 200$. Old prices $(p_x, p_y) = (2, 2)$; new prices $(2, 3)$. Find the equivalent variation A and compensating variation B.

(A) $A = 30, B = 70$

(B) $A = 40, B = 50$

(C) $A = 50, B = 75$

(D) $A = 60, B = 60$

Question 9

Topic: Optimization **Difficulty:** Easy **Status:** Ready

$U(x, y) = -\left[(10 - x)^2 + (10 - y)^2\right]$. All prices equal 1, income = 40. Optimal (x, y)?

(A) $(10, 10)$

(B) $(0, 0)$

(C) $(5, 5)$

(D) None of these

Question 10

Topic: Production Theory **Difficulty:** Moderate **Status:** Ready

$F(K, L) = \min\{aK, bL\}$, $a, b > 0$, $a \neq b$. For fixed $\bar{K}$, the marginal product of labor is

(A) 0

(B) $1/a$ if $L < (a/b)\bar{K}$ and 0 otherwise

(C) $1/b$ if $L < (b/a)\bar{K}$ and 0 if $L > (b/a)\bar{K}$

(D) None of the above

Question 11

Topic: Consumer Theory **Difficulty:** Moderate **Status:** Ready

Let $e_i(p_0)$ be the price elasticity of demand for good X of consumer i at p_0, with heterogeneous quantities. What is the elasticity of the aggregate demand at p_0?

(A) $\sum e_i(p_0)$

(B) $\sum \frac{q_i(p_0)}{\sum_j q_j(p_0)} e_i(p_0)$

(C) $\sum \frac{1}{N} e_i(p_0)$

(D) None of these

Question 12

Topic: Market Structure **Difficulty:** Moderate **Status:** Ready

m identical firms with $C(q) = q^2 + 1$. Industry demand $D(P) = a - bP$, $a, b > 0$. Short-run equilibrium output per firm?

(A) 0

(B) $a/(m + 2b)$

(C) $a/(m^2 + b)$

(D) $a/(m + b/2)$

Question 13

Topic: Market Structure **Difficulty:** Moderate **Status:** Ready

$C(q) = 3q^2 + 800$; $P = 280 - 4q$. Find the price elasticity of demand at the profit-maximizing price.

(A) -4.5

(B) -3.5

(C) -2.5

(D) -1.5

Question 14

Topic: Macroeconomics **Difficulty:** Easy **Status:** Ready

Solow model with saving s, depreciation δ, labor growth n, no technological progress. Steady-state capital–output ratio?

(A) $s/(n+\delta)$

(B) $n/(\delta+n)$

(C) $\delta/(s+n)$

(D) $1/(s+n+\delta)$

Question 15

Topic: Combinatorics **Difficulty:** Easy **Status:** Ready

Number of arrangements of the word "PANDEMIC" (8 distinct letters) such that the vowels appear together.

(A) $6 \times (3!)(5!)$

(B) $5 \times (3!)(5!)$

(C) $4 \times (3!)(5!)$

(D) $1 \times (3!)(5!)$

Question 16

Topic: Algebra / Functions **Difficulty:** Moderate **Status:** Ready

$f(x) = x^2 - x - 1$, $g(x) = x + 1$. Let $\alpha_1 > 0$ and $\alpha_2 < 0$ be the roots of $f(x) = 0$; let $\beta_1 > 0$ and $\beta_2 < 0$ be the roots of $f(g(x)) = 0$. Identify which statement is *incorrect*.

(A) $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 1$

(B) $\alpha_1 + \beta_2 = \alpha_2 + \beta_1 = 0.5$ [as printed]

(C) $\alpha_1 + \beta_1 = -(\alpha_2 + \beta_2) = \sqrt{5}$

(D) $\alpha_1 + \alpha_2 = -(\beta_1 + \beta_2) = -1$

Question 17

Topic: Functions **Difficulty:** Easy **Status:** Ready

$f(x) = \sqrt{1-x}$. Then $f(1-f(x))$ equals

(A) x

(B) $\sqrt{1-x}$

(C) x^2

(D) $1-x^2$

Question 18

Topic: Calculus **Difficulty:** Moderate **Status:** Ready

f is increasing, concave, C^2; g is decreasing, convex, C^2. Then $G(x) = g(f(x))$ is

(A) increasing and convex

(B) decreasing and convex

(C) increasing and concave

(D) decreasing and concave

Question 19

Topic: Calculus **Difficulty:** Moderate **Status:** Ready

$f : \mathbb{R} \to \mathbb{R}$ is C^2 with $f''(x) > 0$ for all x, $f(1) = 1$, $f(2) = 2$. Then,

(A) $0 < f'(2) < 1$

(B) $f'(2) > 1$

(C) $f'(2) = 1$

(D) $f'(2) = 0$

Question 20

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

A, B non-singular of the same order, $C = BAB^{-1}$. For scalar λ, $\det(C + \lambda I)$ equals

(A) $\det A$

(B) $\det B$

(C) $\det(A + \lambda I)$

(D) $\det(B + \lambda I)$

Question 21

Topic: Calculus **Difficulty:** Easy **Status:** Ready

$\lim_{x \to e} \frac{\ln x - 1}{x - e}$.

(A) 0

(B) e

(C) $1/e$

(D) None of these

Question 22

Topic: Calculus **Difficulty:** Moderate **Status:** Ready

$f : [0, \infty) \to \mathbb{R}$, $f(0) = 0$, $f''(x) > 0$ for $x > 0$. Then $f(x)/x$ is

(A) increasing in $(0, \infty)$

(B) decreasing in $(0, \infty)$

(C) increasing in $(0, 1]$ and decreasing in $(1, \infty)$

(D) decreasing in $(0, 1]$ and increasing in $(1, \infty)$

Question 23

Topic: Combinatorics **Difficulty:** Easy **Status:** Ready

$f : A \to B$ with $|A| = 5$, $|B| = 2$. How many onto functions?

(A) $5^2 - 1$

(B) $5^2 - 2$

(C) $2^5 - 1$

(D) $2^5 - 2$

Question 24

Topic: Functions **Difficulty:** Easy **Status:** Ready

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{x}{1+x^2}$. Then

(A) $-1 \leq f(x) \leq 1$

(B) $-1 \leq f(x) \leq 1/2$

(C) $-1/2 \leq f(x) \leq 1$

(D) $-1/2 \leq f(x) \leq 1/2$

Question 25

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

A is 3×3 with rank 3; B is 3×4 with rank 3. Then rank(AB) is

(A) 3

(B) 4

(C) 6

(D) 7

Question 26

Topic: Linear Algebra **Difficulty:** Moderate **Status:** Needs Review

Review note: answer option could not be parsed confidently; source status is Needs Review.

Let A be a specified 6×8 (or square, source unclear) matrix [the entries of A were not legibly transcribed in the source]. Which one is an eigenvalue of A?

(A) 2

(B) 1

(C) 3

(D) 5

Question 27

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

A is a 5×5 non-null singular matrix. Then $Ax = 0$ has

(A) only the trivial solution

(B) exactly 5 solutions

(C) no solution

(D) infinitely many solutions

Question 28

Topic: Probability **Difficulty:** Easy **Status:** Ready

A family has two children. Find the probability that both are boys given that at least one is a boy.

(A) 1/2

(B) 2/3

(C) 1/3

(D) 1/4

Question 29

Topic: Probability **Difficulty:** Easy **Status:** Ready

Box 1 contains {1 black, 1 white}; Box 2 contains {1 black, 2 white}. Select a box at random and then a ball at random. Probability that the drawn ball is black?

(A) 5/12

(B) 2/5

(C) 1/6

(D) 5/11

Question 30

Topic: Functions **Difficulty:** Easy **Status:** Ready

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = (x^2 + 1)^{2022}$.

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto