

Statstrive ExamIQ

ISI MSQE PEA 2024 Question Paper

Regenerated from the local solution TeX source. Items missing full source detail are marked Needs Review.

Question	Topic	Website Status
1	Calculus	Ready
2	Sequences and Series	Ready
3	Calculus	Ready
4	Functions	Ready
5	Functions	Ready
6	Calculus	Ready
7	Calculus	Ready
8	Linear Algebra	Ready
9	Linear Algebra	Ready
10	Linear Algebra	Ready
11	Linear Algebra	Ready
12	Combinatorics	Ready
13	Combinatorics	Ready
14	Combinatorics	Ready
15	Probability	Ready
16	Statistics	Ready
17	Probability	Ready
18	Probability	Ready
19	Expected Value and Variance	Ready
20	Distributions	Ready
21	Growth Models	Ready
22	Growth Models	Ready
23	Growth Models	Ready
24	Growth Models	Ready
25	International Trade / General Equilibrium	Ready
26	International Trade / General Equilibrium	Ready
27	Consumer Theory	Ready
28	Intertemporal Optimization	Ready
29	Market Structure	Ready
30	Consumer Theory	Needs Review

Contents

Question 1

Topic: Calculus **Difficulty:** Moderate **Status:** Ready

If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function at $a \in \mathbb{R}$ such that $f'(a) = af(a)$, then what is

$$\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a} ?$$

- (A) $af(a)$
- (B) $f(a)$
- (C) $(1 - a^2)f(a)$
- (D) None of the previous options

Question 2

Topic: Sequences and Series **Difficulty:** Easy **Status:** Ready

Suppose S_n is defined as follows for every positive integer $n \geq 2$:

$$S_n = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{n^2}\right).$$

The value of $\lim_{n \rightarrow \infty} S_n$ is

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) ∞

Question 3

Topic: Calculus **Difficulty:** Moderate **Status:** Ready

Suppose

$$\lim_{x \rightarrow 0} \frac{e^{a_1 x} - 1}{a_2 x^2 + a_3 x} = 1,$$

where a_1, a_2, a_3 are given real numbers. Then it is necessarily true that

- (A) $a_1 = a_2 = a_3 = 1$
- (B) $a_1 = a_3 \neq 0$
- (C) $a_2 = 0$
- (D) $a_2 + a_3 \neq 0$

Question 4

Topic: Functions **Difficulty:** Easy-Moderate **Status:** Ready

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} cx^2 + ax + b, & x < 0, \\ bx^2 + cx + a, & 0 \leq x < 2, \\ ax^2 + bx + c, & x \geq 2, \end{cases}$$

where a, b, c are positive real numbers. Which of the following statements is correct, under the assumption that f is continuous?

- (A) f is continuous for all values of a, b, c
- (B) f is continuous iff $a - b = b - c$
- (C) f is continuous iff $a = b$ and $c = 2a$
- (D) f is continuous iff $a = b = c$

Question 5

Topic: Functions **Difficulty:** Moderate **Status:** Ready

Consider $f : [0, 1] \rightarrow [0, 1]$ such that $f(x) = \frac{x}{2-x}$. Which of the following statements is *incorrect*?

- (A) $f(0) = 0$ and $f(1) = 1$
- (B) $f(1 - f(x)) = 1 - x$
- (C) f is strictly concave in the interval $(0, 1)$
- (D) f is strictly increasing in the interval $(0, 1)$

Question 6

Topic: Calculus **Difficulty:** Easy **Status:** Ready

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function $f(x) = |x| + x^2 \forall x \in \mathbb{R}$. Which of the following statements about f is correct?

- (A) f is differentiable
- (B) f is concave but not differentiable
- (C) f is convex but not differentiable
- (D) f is discontinuous

Question 7**Topic:** Calculus **Difficulty:** Easy **Status:** Ready $\int x^3 e^{x^2} dx$ equals

(A) $\frac{x(x-1)}{2} e^{x^2}$

(B) $\frac{x^2-1}{2} e^{x^2}$

(C) $\frac{x(x+1)}{2} e^{x^2}$

(D) $\frac{x^2+1}{2} e^{x^2}$

Question 8

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

Let A be a 3×3 matrix having eigenvalues 2, 7, 5. What is the determinant of $A + 2I$?

- (A) 252
- (B) 70
- (C) 420
- (D) 84

Question 9

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

Let x and y be two column vectors of length 3 such that $\sum_i x_i y_i = 1$. What is the rank of xy^T ?

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Question 10

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

Let A be a 3×3 matrix such that $Ax = x$ for all column vectors x of length 3. Which of the following statements is correct?

(A) No such A exists

(B) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(C) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(D) A exists but is different from the matrices given in parts (B) and (C)

Question 11

Topic: Linear Algebra **Difficulty:** Easy **Status:** Ready

Let $S = \{(x_1, 0) : x_1 \in \mathbb{R}\}$ and $T = \{(0, x_2) : x_2 \in \mathbb{R}\}$. Which of the following is correct?

- (A) S, T and $S \cap T$ are vector spaces
- (B) S, T and $S \cup T$ are vector spaces
- (C) $S \cup T$ and $S \cap T$ are vector spaces
- (D) Neither $S \cup T$ nor $S \cap T$ is a vector space

Question 12

Topic: Combinatorics **Difficulty:** Easy **Status:** Ready

In a chess tournament, there are both boys and girls. Each player plays with another player exactly once. If there are 45 games in total and exactly 15 of them feature only boys, how many games feature a boy and a girl?

- (A) 6
- (B) 15
- (C) 20
- (D) 24

Question 13

Topic: Combinatorics **Difficulty:** Easy **Status:** Ready

What is the number of possible arrangements of the letters of the word 'madam' such that the two 'a's never appear in consecutive positions?

- (A) 30
- (B) 24
- (C) 18
- (D) 12

Question 14

Topic: Combinatorics **Difficulty:** Easy **Status:** Ready

A monkey starts at $(0, 0)$ on the xy -plane in period 1. From any position (x, y) in a period, the monkey can only jump to (a, b) in the next period, where $a \in \{x + 1, x, x - 1\}$ and $b \in \{y + 1, y, y - 1\}$. How many possible positions can the monkey be in period 2?

- (A) 9
- (B) 6
- (C) 4
- (D) 3

Question 15

Topic: Probability **Difficulty:** Easy **Status:** Ready

In the previous question, suppose in every period the monkey can go to any of the possible positions in the next period with equal probability. Then what is the probability that the monkey is at a distance of more than 1 from $(0, 0)$ in period 2?

- (A) 0
- (B) $\frac{16}{81}$
- (C) $\frac{1}{3}$
- (D) $\frac{4}{9}$

Question 16

Topic: Statistics **Difficulty:** Easy **Status:** Ready

For a given data set, let the least squares regression line be $y = 10 + 2x$. It is given that variance of x is 9 and variance of y is 81. What is the correlation coefficient between x and y ?

- (A) $\frac{1}{3}$
- (B) $\frac{1}{2}$
- (C) $\frac{2}{3}$
- (D) $\frac{3}{4}$

Question 17

Topic: Probability **Difficulty:** Easy-Moderate **Status:** Ready

An urn contains 4 white, 6 red, and 5 black balls. 5 balls are randomly selected from the urn. Let X and Y denote respectively the number of white and black balls selected. Suppose $Y = 2$, i.e., 2 of the 5 selected balls are black. What is the probability that X takes the value 2?

- (A) $\frac{3}{10}$
- (B) $\frac{4}{10}$
- (C) $\frac{3}{15}$
- (D) $\frac{4}{15}$

Question 18

Topic: Probability **Difficulty:** Easy **Status:** Ready

In answering an MCQ with 4 choices, a student either knows the answer (with probability $\frac{1}{4}$) or guesses (with probability $\frac{3}{4}$). A guess is correct with probability $\frac{1}{4}$. What is the probability that the student knew the answer, given that he answered correctly?

- (A) $\frac{3}{7}$
- (B) $\frac{4}{7}$
- (C) $\frac{3}{4}$
- (D) 1

Question 19

Topic: Expected Value and Variance **Difficulty:** Easy **Status:** Ready

Let X_1, X_2, \dots, X_5 be i.i.d. random variables with $E(X_i) = 0$ and $V(X_i) = 1$. What is the value of

$$E \left[\frac{(\sum_{i=1}^5 X_i)^2}{5} \right] ?$$

- (A) 0
- (B) 1
- (C) -1
- (D) None of the previous options

Question 20

Topic: Distributions **Difficulty:** Moderate **Status:** Ready

Consider $X \sim \text{Bin}(n, p)$. Which of the following is *incorrect*?

- (A) If np is an integer, then the mean and mode of this distribution coincide
- (B) If n is even and $p = \frac{1}{2}$, then the median of this distribution is $n/2$
- (C) If $P(X = k) = P(X = n - k)$ for every $k \in \{0, 1, \dots, n\}$, then $p = \frac{1}{2}$
- (D) If $(n + 1)p - 1$ is an integer, then this distribution is unimodal

Question 21

Topic: Growth Models **Difficulty:** Moderate **Status:** Ready

Consider an economy with $y = Ak$ (per-capita form), saving rate $s \in (0, 1)$, population growth rate $n > 0$ and depreciation rate $\delta \in (0, 1)$. Assume parameters ensure positive long-run growth of y . Which of the following is *incorrect*?

- (A) The growth rate of capital–labour ratio is $sA - (n + \delta)$
- (B) If investment $I > \delta K$, then output grows at a positive rate
- (C) The economy is always on the steady state growth path
- (D) Increasing the savings rate s will not have any effect on the long-run growth rate of output per worker

Question 22

Topic: Growth Models **Difficulty:** Easy **Status:** Ready

Solow economy with $Y = K^{1/3}L^{2/3}$, no population growth or technological change, $\delta = 0.05$, $s = 0.2$.
What is the steady-state capital-labour ratio?

- (A) 6
- (B) 8
- (C) 2
- (D) 4

Question 23

Topic: Growth Models **Difficulty:** Moderate **Status:** Ready

With the same Solow economy (no population growth, $\delta = 0.05$, $Y = K^{1/3}L^{2/3}$), a social planner wishes to maximize steady-state per-capita consumption. What savings rate s achieves this?

- (A) $\frac{2}{3}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{3}$
- (D) $\frac{1}{4}$

Question 24

Topic: Growth Models **Difficulty:** Moderate-Hard **Status:** Ready

A Solow economy has $y = k^{1/2}$, $\delta = 0$, labour grows at $n > 0$. If the steady-state value of k is 50 and the current value of k is 2, what is the current growth rate of per-capita output?

- (A) $24n$
- (B) $25n$
- (C) 0.3
- (D) None of the previous options

Question 25

Topic: International Trade / General Equilibrium **Difficulty:** Moderate **Status:** Ready

Two countries A and B produce wheat and television respectively, with $Y_A = 200$ and $Y_B = 250$. The TV price is normalised to 1, the relative price of wheat is p . Let E_A (in wheat units) and E_B (in TV units) denote total expenditures. The income identity is $pY_A + Y_B = pE_A + E_B$. Initially $E_A = 100$; A reduces E_A to 70. Suppose $1/3$ of each country's expenditure is on wheat. What happens to the equilibrium p ?

- (A) p stays the same
- (B) p increases
- (C) p decreases
- (D) The effect is ambiguous

Question 26

Topic: International Trade / General Equilibrium **Difficulty:** Hard **Status:** Ready

Same setup as Q25, except now *each* country spends $2/3$ of its expenditure on its *own* good and $1/3$ on the foreign good. What is the impact on p when E_A falls from 100 to 70?

- (A) p stays the same
- (B) p increases
- (C) p decreases
- (D) The effect is ambiguous

Question 27

Topic: Consumer Theory **Difficulty:** Moderate **Status:** Ready

A consumer consumes two goods X and Y . It is observed that her consumption of X always falls when the price of X falls, *ceteris paribus*. Suppose now income rises, with prices of X and Y held constant. What happens to consumption of X ?

- (A) Consumption of X falls
- (B) Consumption of X rises
- (C) Consumption of X remains unchanged
- (D) Indeterminate: consumption of X could rise, fall, or remain unchanged

Question 28

Topic: Intertemporal Optimization **Difficulty:** Easy-Moderate **Status:** Ready

A has a pond which yields f fish at $t = 1$ and nothing at $t = 2$. He consumes only fish. Storage is costless and undepreciated; no discounting. Utility in any period t is $u(c_t) = c_t^n$ with $0 < n < 1$. Optimal consumption?

- (A) Any division is optimal
- (B) $c_1 = c_2 = f/2$
- (C) $c_1 = f, c_2 = 0$
- (D) $c_1 = 0, c_2 = f$

Question 29

Topic: Market Structure **Difficulty:** Hard **Status:** Ready

Country C has 10,000 identical farmers with cost $C(q_i) = 0.5q_i^2 + 4q_i + 100$, aggregate demand $Q = -10,000p + 400,000$. Current price support $p = 30$; government buys all unsold output at 30. The government proposes removing the support and giving each farmer an equal share of the saved expenditure as a lump-sum transfer. Each farmer must, however, pay 300 to open a bank account to receive the transfer. By how much does a farmer gain (or lose)?

- (A) Rise by 8
- (B) Rise by 4
- (C) Fall by 4
- (D) Fall by 8

Question 30

Topic: Consumer Theory **Difficulty:** Hard **Status:** Needs Review

Review note: source status is Draft.

A consumer consumes three goods X, Y, Z over three periods. Prices and bundles:

$$\text{Period 1: } p^1 = (2, 3, 3), x^1 = (3, 1, 7),$$

$$\text{Period 2: } p^2 = (3, 2, 3), x^2 = (7, 3, 1),$$

$$\text{Period 3: } p^3 = (3, 3, 2), x^3 = (1, 7, 3).$$

Which of the following statements about her preferences is correct?

- (A) Preferences are not complete
- (B) Preferences are transitive though not necessarily complete
- (C) Preferences are incomplete and intransitive
- (D) Preferences are complete and transitive