ISI MSQE 2026 — PEA Question Paper (Typed)

1. Suppose a and b are positive integers that are relatively prime. It is known that:

$$\frac{a}{b} = \sum_{n=1}^{2026} \frac{1}{n^2 + 15n + 56}$$

Then the value of $a + b$ is
(A) 9149 (B) 9229 (C) 8189 (D) 11

2. If $x^2 - 3x + 1 = 0$, then the value of $\left(x^4 + \frac{1}{x^4}\right)$ is
(A) 43 (B) 45 (C) 47 (D) 49

3. Let $f : \mathbb{R} \to \mathbb{R}$ such that $f(x^3 + 1) = x^6 + 4x^3 + 8$ for all $x \in \mathbb{R}$. Then the value of

$$\int_1^2 f(x)\, dx$$

equals
(A) $\frac{29}{3}$ (B) 10 (C) 11 (D) $\frac{31}{3}$

4. Determine the value of the given integral:

$$\int_0^2 |x^2 - 7x + 10|\, dx$$

(A) 8 (B) 9 (C) $\frac{26}{3}$ (D) $-\frac{23}{3}$

5. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that

$$\int_0^1 f(t)\, dt = 1,$$

and let

$$P(x) = \sum_{k=1}^{n} a_k x^k, \quad \text{with} \sum_{k=1}^{n} a_k = 1.$$

Then there exists $c \in (0, 1)$ such that
(A) $f(c) = 2\sum_{k=1}^{n} a_k$ (B) $f(c) = \int_0^1 P(t)\, dt$ (C) $f(c) = P'(c)$ (D) $f(c) = P(c)$

6. Let $f : [0, \pi/4] \to \mathbb{R}$ be continuous. Then there exists $c \in (0, \pi/4)$ such that
(A) $f(c) = \int_0^{\pi/4} f(t)\, dt$
(B) $f(c) = 2\cos(2c) \int_0^{\pi/4} f(t)\, dt$
(C) $f(c) = \sin(2c)$
(D) $f(c) = \sin(2c) \int_0^{\pi/4} f(t)\, dt$

7. Suppose $f : [a, b] \to \mathbb{R}$ is continuous and satisfies

$$\int_a^x f(t)\,dt = \int_x^b f(t)\,dt \quad \text{for all } x \in [a, b].$$

Then $f(x)$ must be

(A) A non-zero constant (B) A linear function (C) Identically zero (D) An odd function about $\frac{a+b}{2}$

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous odd function, vanishing exactly at one point and satisfying $f(1) = \frac{1}{2}$. If

$$\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14},$$

where

$$F(x) = \int_{-1}^x f(t)\,dt, \qquad G(x) = \int_{-1}^x t\,|f(f(t))|\,dt,$$

then the value of $f\left(\frac{1}{2}\right)$ is

(A) $\frac{1}{7}$ (B) 7 (C) -7 (D) $\frac{7}{2}$

9. If the correlation between X and Y is zero, then which of the following is always true?

(A) They are independent

(B) There is no linear relationship

(C) They cannot be functionally related

(D) All of the above

10. In simple linear regression $Y = a + bX$, the least squares line based on n data points always passes through

(A) $(0, 0)$ (B) $\left(\frac{1}{n}, \frac{1}{n}\right)$ (C) $(1, 1)$ (D) None of the previous

11. Given sample size n, reducing the significance level α

(A) Increases Type I error (B) Decreases Type II error (C) Increases power (D) Decreases power

12. If $X \sim \text{Bernoulli}(p)$, then $E(X^2)$ equals

(A) p^2 (B) p (C) $p(1-p)$ (D) $p + p^2$

13. For $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$, which is unbiased for λ^2?

(A) $\bar{X}^2$ (B) $\bar{X}$ (C) $\bar{X}^2 - \frac{\bar{X}}{n}$ (D) $\bar{X}^2 - \bar{X}$

14. A dataset consists of 9 observations arranged in increasing order: $x_1 \le x_2 \le \cdots \le x_9$. It is known that the mean of the data is 20 and the median is 18. If the largest observation x_9 is replaced by a very large number, which of the following statements must be true?

(A) Both the mean and the median increase

(B) The mean increases but the median remains unchanged
(C) The median increases but the mean remains unchanged
(D) Both the mean and the median remain unchanged

15. A restaurant offers two types of meals, vegetarian and non-vegetarian. 60% of customers order vegetarian meals while 40% order non-vegetarian meals. Among vegetarian customers, 30% order dessert while 50% order dessert among non-vegetarian customers. If a randomly selected customer ordered dessert, what is the probability that the customer ordered a vegetarian meal?

(A) 0.375 (B) 0.474 (C) 0.600 (D) 0.750

16. Suppose an agent's preferences can be represented by a utility function $u(x, y) = x - \ln y$, where $x > 0$ and $y > 0$ are the amounts of the two goods consumed. Then the underlying preference relation must be

(A) Incomplete
(B) Complete but intransitive
(C) Complete and transitive but discontinuous
(D) Complete, transitive and continuous but not convex

17. Suppose there are two agents, A and B, whose utilities are given respectively by

$$u_A = x_A + y_A - m(x_A - y_A)^2 \quad \text{and} \quad u_B = x_B + y_B,$$

where x_i and y_i are respectively the amounts of two goods X and Y consumed by agent $i \in \{A, B\}$. The total endowments of the two goods are x and y. Suppose initially the entire endowment of Y is owned by A, while the entire endowment of X is owned by B. Then A will voluntarily transfer a positive amount of Y to B if

(A) $m > 0$ (B) $m < 0$ (C) $y > 2$ (D) $my > \frac{1}{2}$

18. Suppose there are two agents, A and B, whose utilities are given respectively by $u_A = x_A + y$ and $u_B = 2x_B + y$, where x_A and x_B are respectively the amounts consumed of a private good, while y is the amount consumed of a public good. The production in this economy are given by $y + m(x_A + x_B) = 1$, where $m \in (0, 1)$. Suppose a social planner decides that 1 unit should be produced in this economy. For what value of m will the resulting allocation be Pareto optimal?

(A) $\frac{4}{5}$ (B) $\frac{4}{7}$ (C) $\frac{4}{9}$ (D) $\frac{4}{11}$

19. Suppose there are three consumers, A, B, and C. Each wishes to purchase one unit of a good. The consumers' valuations are v_A, v_B, and v_C, respectively, with $\frac{7}{6}v_B > v_A > v_B > \frac{8}{5}v_C > 0$. The market is served by a monopolist who produces at zero marginal cost. The monopolist knows the values of v_A, v_B, and v_C, but cannot tell the consumers apart. The monopolist would maximize profit by charging a price per unit

(A) $= v_A$ (B) $= v_B$ (C) $= v_C$ (D) $\in (v_C, v_B)$

20. Suppose there are n agents. All agents $i \in \{1, \ldots, n\}$ simultaneously choose their private consumption x_i and expenditure on a public good y_i, according to preferences

represented by the utility function $u_i = x_i \left(\sum_{j=1}^{n} y_j \right)$, subject to the budget constraint $b = x_i + y_i$, with $b > 0$ and $x_i, y_i \geq 0$ for all i. Then, as n increases, the total Nash equilibrium expenditure on the public good

(A) Remains invariant

(B) Monotonically increases and approaches b

(C) Approaches $\frac{b}{n}$

(D) Monotonically decreases and approaches 0

21. There has been heavy snowfall in a city. A and B are neighbours in some neighbourhood of the city. It is impossible for a snow plough to clear the street in front of A's house without clearing it in front of B's. A's inverse demand for snow ploughing services is $12 - q$, where q is the number of times a street is ploughed, while B's inverse demand is $8 - q$. The marginal cost of getting the street ploughed is 16. What is the efficient level of provisioning of snow ploughing services, viewed as a public good?

(A) 6 (B) 4 (C) 2 (D) 1

22. Suppose a lump-sum tax of 1 rupee has to be paid by the purchaser for each unit bought of a good traded under perfectly competitive conditions. Imposition of this tax leads to an 80 paisa increase in the price paid by the purchasers. Then

(A) Demand must be perfectly elastic

(B) Demand must be perfectly inelastic

(C) Demand and supply elasticities must both be positive but finite in absolute value

(D) Supply must be perfectly elastic

23. A consumer consumes two commodities X and Y, and always spends one-fourth of their income on X. Suppose the income elasticity of demand for X is 5. Is Y inferior?

(A) Yes (B) No

(C) Y is an inferior good if and only if the price of Y is at least twice the price of X

(D) Y is an inferior good if and only if the price of Y is at most half the price of X

24. If half of the total quantity demanded of some good is purchased by 75 consumers, each of whom has a price elasticity of demand for the good equalling 2, and the other half is purchased by 25 consumers, each of whom has a price elasticity of demand for the good equalling 3, what is the price elasticity of demand for the good of the 100 consumers taken together?

(A) Cannot be determined (B) 2.25 (C) 2.5 (D) 2.75

25. There are two countries, H and F. Each produces a single good, with the output of the good produced by country i denoted by Y^i, $i \in \{H, F\}$. Let the good produced by H be the numeraire, and let p denote the relative price of the good produced by F. Consumers in both countries consume both goods, and only those goods. Suppose utilities are Cobb–Douglas and consumers in country i spend $\frac{1}{4}$ fraction of their total expenditure on the good produced by country j, $i, j \in \{H, F\}$, $i \neq j$. Suppose E^i is the total expenditure of consumers of country i measured in terms of the good produced

by country i, and total income equals total expenditure for the two countries taken together. It follows that

$$Y^H + pY^F = E^H + pE^F.$$

If all markets clear, $Y^H = 120$, $Y^F = 100$, and $E^H = 80$, then the equilibrium price p is

(A) 8 (B) 6 (C) 4 (D) 2

The following information is for Question numbers 26 and 27: An economy produces a single good using a single factor of production, labour, with one unit of labour required to produce one unit of the good. Let output of the good be denoted by Y, and labour employed in production be denoted by L. Let P be the price of the good and W be the wage rate for labour, and assume that the market for the good is perfectly competitive and always clears in equilibrium. There is a single asset, money, and an aggregate consumer whose consumption of the good is denoted by C and money holding is denoted by M. Suppose the consumer maximizes the utility function

$$U\left(C, \frac{M}{P}\right) = \frac{3}{4}\ln C + \frac{1}{4}\ln\left(\frac{M}{P}\right).$$

The consumer has an initial endowment of money, $\bar{M} = 100$, and sells its labour to provide for consumption and money holding. Its total labour endowment is 50. Assume money supply is constant, i.e. $M = \bar{M}$.

26. Suppose the price is flexible, so that labour is fully employed. Then the equilibrium price is

(A) 2 (B) 4 (C) 6 (D) 8

27. Consider an alternate scenario, where the price is fixed at $P = 10$. Then the equilibrium output is

(A) 15 (B) 20 (C) 25 (D) 30

The following information is for Question numbers 28 and 29: There is a Solow type economy with an aggregate production function $Y = K^{1/2}L^{1/2}$, where Y is output, L is labour and K is capital employed, with labour and capital fully employed. Suppose there is no depreciation, the growth rate of labour n is 0.02, and the savings rate s is positive. Denote the capital–labour ratio $\frac{K}{L}$ at the current date by k, the steady-state capital–labour ratio by k^*, and the per capita output $\frac{Y}{L}$ by y.

28. If $k^* = 9$, and $k = 1$ at the current date, then the growth rate of y at the current date is

(A) 0.02 (B) 0.04 (C) 0.03 (D) Indeterminable

29. Two economies A and B, with the same production function, depreciation, $n = 0.02$, and $s > 0$. Denoting capital–labour ratios at the current date by k_i, and steady-state capital–labour ratios by k_i^*, $i \in \{A, B\}$, let $k_A = 3$, $k_B = 4$, $k_A^* = 9$ and $k_B^* = 16$. Then y at the current date is growing

(A) Faster in A than in B

(B) Faster in B than in A

(C) At the same rate in A and B

(D) At an indeterminable relative rate

30. A person lives for two periods. They earn a wage $w > 0$ in period 1, part of which they consume in period 1, denoted C_1, saving the rest, denoted S. They earn nothing in period 2, so their period 2 consumption, denoted C_2, is supported by their savings made in period 1 and the interest accrued on that savings, where $r > 0$ is the net interest rate. The person maximizes the utility function given by:

$$U(C_1, C_2) = \log C_1 + \frac{1}{1+\rho} \log C_2,$$

where $\rho > 0$ is the rate of time preference. Suppose w increases by 1% and r decreases by 1%. Then S

(A) Increases by more than 1%

(B) Increases by 1%

(C) Decreases by 1%

(D) Decreases by more than 1%

ISI MSQE — PEB (Part B) Subjective

edusure.in — 8100824600 / 8100824601 — contact@edusure.in

1. For any natural number m, let

$$m! = m \times (m-1) \times \cdots \times 2 \times 1.$$

Let $(a_k)_{k=1}^{\infty}$ be a sequence of positive real numbers. Suppose that

$$\sum_{k=1}^{\infty} \frac{a_{k+1}}{a_k} \leq 1.$$

Show that for all natural numbers $n \geq 2$,

$$a_n \leq \frac{1}{(n-1)!}.$$

2. Consider for each $t > 0$, the system of two linear equations in two variables:

$$(1+t^2)x + ty = 0,$$

$$t^2 x + e^{-t} y = 0.$$

Show that there exists a number $t_0 > 0$ such that at $t = t_0$, the set of solutions (x, y) of the above set of equations will form a one-dimensional subspace of $\mathbb{R}^2$.

3. Let $T > 0$. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function with period T, that is,

$$f(x+T) = f(x),$$

for all $x \in \mathbb{R}$. Show that f is constant.

4. An experiment has N outcomes, say $\omega_1, \omega_2, \ldots, \omega_N$. Suppose that ω_{j+1} is twice as likely as ω_j, where $j \in \{1, 2, \ldots, N-1\}$.

Let

$$A_k = \{\omega_1, \omega_2, \ldots, \omega_k\}.$$

Find $P(A_k)$.

5. There are M students of which K are female and $M-K$ are male, $K \in \{1, \ldots, M-1\}$. A debate committee comprising of $n \in \{1, \ldots, M-1\}$ students has to be formed.

An urn with the unique roll numbers of all students is created to ensure that the selection process is randomized and unbiased. Roll numbers are drawn one by one from the urn following the rule that once a roll number is selected, it is withdrawn permanently from the urn.

(a) Find the probability that the jth number drawn from the urn is that of a female student given that k female students have already been selected into the committee.

(b) Find the probability that the jth number drawn from the urn is that of a female student given that there are a total of k female students in the committee.

6. A profit maximizing firm possesses a linear technology which can convert 1 unit of labour input into 1 unit of output. Labour is available at wage $w > 0$ per unit.

The firm, a monopoly, faces an inverse demand function $p(q)$, with $p(0) > w$, $\lim_{q\to\infty} p(q) = 0$, $\lim_{q\to\infty} qp(q) = 0$, $p'(q) < 0$, and $p''(q) \leq 0$.

The firm first hires labour, then executes production, following which it sells output to consumers and collects revenue.

(a) Suppose labour can be paid wages after the collection of revenue. What is the firm's optimal output?

(b) Suppose now labour must be paid wages prior to the collection of revenue.

(i) Suppose further that the firm has no cash in hand to pay wages upfront, but can borrow an amount B prior to production to pay wages upfront, with repayment due after the collection of revenue. The net interest rate on any such loan is $r > 0$. What is the firm's optimal output?

(ii) Suppose instead further that the firm cannot borrow, but it has cash $C \geq 0$ available with it which can be used to pay wages upfront. What is the firm's optimal output?

7. A household lives for two periods, $t = 1, 2$. In each period it chooses consumption C_t and labor supply N_t. It also chooses savings S in period 1, which pays gross return $(1 + r)$ in period 2.

The household earns wage income $w_t N_t$ and receives government transfers T_t in period t, where w is wage rate.

Preferences are

$$U = \ln C_1 + \ln(1 - N_1) + \beta\big[\ln C_2 + \ln(1 - N_2)\big], \qquad \beta \in (0, 1).$$

The household faces budget constraints

$$C_1 + S = w_1 N_1 + T_1, \qquad C_2 = (1 + r)S + w_2 N_2 + T_2.$$

Assume the government satisfies the present-value transfer constraint

$$\bar{T} = T_1 + \frac{T_2}{1+r},$$

where $\bar{T}$ is fixed.

(a) Solve for the household's optimal consumption allocations (C_1, C_2).

(b) Does optimal consumption depend on the timing of transfers (T_1, T_2), or only on their present value $\bar{T}$?

(c) Now suppose the household evaluates period-2 utility with an additional present-bias factor $\delta \in (0, 1)$:

$$U = \ln C_1 + \ln(1 - N_1) + \delta\beta\big[\ln C_2 + \ln(1 - N_2)\big].$$

The household cannot commit: in period 1 it chooses (C_1, N_1, S), and in period 2 it chooses (C_2, N_2) taking savings S as given. Solve for the optimal consumption allocations (C_1, C_2).