ISI MSQE Solutions

Paper: PEA
Year/Batch: 2026
Prepared for: Statstrive

ISI MSQE 2026 — PEA Question Paper: Polished Solutions

This document presents complete, exam-quality solutions to all 30 multiple-choice questions of the ISI MSQE 2026 PEA paper. Each solution is derived from first principles using standard tools of calculus, real analysis, probability, statistics, microeconomics and macroeconomics.

Answer Key Summary (PEA)

Question	Topic	Correct Option	Status
1	Sequences and Series (Telescoping)	A	Verified
2	Algebra	C	Verified
3	Calculus (Definite Integrals)	D	Verified
4	Calculus (Definite Integrals)	C	Verified
5	Real Analysis (Rolle / MVT)	C	Verified
6	Real Analysis (Cauchy MVT)	B	Verified
7	Real Analysis (Integrals)	C	Verified
8	Calculus (L'Hôpital, Symmetry)	B	Verified
9	Statistics (Correlation)	B	Verified
10	Regression	D	Verified
11	Hypothesis Testing	D	Verified
12	Random Variables	B	Verified
13	Statistics (Unbiased Estimation)	D	Verified
14	Statistics (Descriptive)	B	Verified
15	Probability (Bayes)	B	Verified
16	Consumer Theory	D	Verified
17	General Equilibrium (Exchange)	A	Draft
18	General Equilibrium (Public Goods)	–	Needs Review
19	Market Structure (Monopoly Pricing)	B	Verified
20	Game Theory (Public Good NE)	B	Verified
21	Public Goods (Samuelson)	C	Verified
22	Tax Incidence	C	Verified
23	Consumer Theory (Engel Aggregation)	A	Verified
24	Consumer Theory (Aggregate Elasticity)	C	Verified
25	International Trade (Cobb–Douglas)	D	Verified
26	Macroeconomics (Classical)	C	Verified
27	Macroeconomics (Fixed Price)	D	Verified
28	Growth (Solow)	A	Verified
29	Growth (Solow, Cross-country)	B	Verified
30	Intertemporal Choice	B	Verified

Topic Summary

Topic Area	Questions
Calculus / Integration	3, 4, 8
Real Analysis (MVT, Rolle)	5, 6, 7
Algebra / Sequences	1, 2
Probability & Random Variables	12, 15
Statistics (Estimation, Descriptive)	9, 13, 14
Regression & Hypothesis Testing	10, 11
Consumer Theory	16, 23, 24
General Equilibrium & Public Goods	17, 18, 21
Market Structure	19
Game Theory	20
Tax Incidence	22
International Trade	25
Macroeconomics & Money	26, 27
Growth Theory	28, 29
Intertemporal Choice	30

Full Polished Solutions

Question 1

Paper: PEA **Year:** 2026 **Topic:** Sequences and Series
Subtopic/Concept: Telescoping sum, partial fractions **Difficulty:** Easy **Status:** Verified

Question

Suppose a and b are positive integers that are relatively prime, and

$$\frac{a}{b} = \sum_{n=1}^{2026} \frac{1}{n^2 + 15n + 56}.$$

Then the value of $a + b$ is

(A) 9149

(B) 9229

(C) 8189

(D) 11

Solution

Factor the denominator:

$$n^2 + 15n + 56 = (n + 7)(n + 8).$$

Partial fractions give

$$\frac{1}{(n + 7)(n + 8)} = \frac{1}{n + 7} - \frac{1}{n + 8}.$$

Hence the sum telescopes:

$$\sum_{n=1}^{2026} \left(\frac{1}{n + 7} - \frac{1}{n + 8} \right) = \frac{1}{8} - \frac{1}{2034} = \frac{2034 - 8}{8 \cdot 2034} = \frac{2026}{16272}.$$

Now simplify $\frac{2026}{16272}$. Since $2026 = 2 \cdot 1013$ and $16272 = 2 \cdot 8136$,

$$\frac{2026}{16272} = \frac{1013}{8136}.$$

We verify $\gcd(1013, 8136) = 1$: $8136 = 2^3 \cdot 3^2 \cdot 113$, and 1013 is prime (no prime factor up to $\lfloor \sqrt{1013} \rfloor = 31$ divides it). Thus the fraction is already in lowest terms with $a = 1013$, $b = 8136$. Therefore

$$a + b = 1013 + 8136 = 9149.$$

Final Answer

$$\boxed{\text{Option A: 9149}}$$

Common Trap / Note

A common error is to take $1/8 - 1/(N+8)$ with $N = 2026$ but then forget to verify coprimality. The fraction $\frac{2026}{16272}$ reduces by a factor of 2 only.

Question 2

Paper: PEA **Year:** 2026 **Topic:** Algebra
Subtopic/Concept: Symmetric expressions in x and $1/x$ **Difficulty:** Easy **Status:** Verified

Question

If $x^2 - 3x + 1 = 0$, then the value of $\left(x^4 + \frac{1}{x^4}\right)$ is

(A) 43

(B) 45

(C) 47

(D) 49

Solution

From $x^2 - 3x + 1 = 0$ (and $x \neq 0$, since the constant term is non-zero), divide by x:

$$x + \frac{1}{x} = 3.$$

Square:

$$x^2 + \frac{1}{x^2} = \left(x + \tfrac{1}{x}\right)^2 - 2 = 9 - 2 = 7.$$

Square again:

$$x^4 + \frac{1}{x^4} = \left(x^2 + \tfrac{1}{x^2}\right)^2 - 2 = 49 - 2 = 47.$$

Final Answer

$$\boxed{\text{Option C: 47}}$$

Common Trap / Note

No major trap beyond standard calculation care.

Question 3

Paper: PEA **Year:** 2026 **Topic:** Calculus
Subtopic/Concept: Definite integrals, substitution **Difficulty:** Easy **Status:** Verified

Question

Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x^3 + 1) = x^6 + 4x^3 + 8$ for all $x \in \mathbb{R}$. Then $\int_1^2 f(x)\,dx$ equals

(A) $\frac{29}{3}$

(B) 10

(C) 11

(D) $\frac{31}{3}$

Solution

Set $u = x^3 + 1$, so $x^3 = u - 1$ and

$$f(u) = (u-1)^2 + 4(u-1) + 8 = u^2 - 2u + 1 + 4u - 4 + 8 = u^2 + 2u + 5.$$

Therefore

$$\int_1^2 f(x)\,dx = \int_1^2 (x^2+2x+5)\,dx = \left[\frac{x^3}{3} + x^2 + 5x\right]_1^2 = \left(\tfrac{8}{3} + 4 + 10\right) - \left(\tfrac{1}{3} + 1 + 5\right) = \tfrac{7}{3} + 8 = \tfrac{31}{3}.$$

Final Answer

$$\boxed{\text{Option D: } \tfrac{31}{3}}$$

Common Trap / Note

Setting $u = x^3 + 1$ is bijective on $\mathbb{R}$, so f is uniquely determined.

Question 4

Paper: PEA **Year:** 2026 **Topic:** Calculus
Subtopic/Concept: Integral of an absolute value **Difficulty:** Easy **Status:** Verified

Question

Determine the value of

$$\int_0^2 |x^2 - 7x + 10|\,dx.$$

(A) 8

(B) 9

(C) $\frac{26}{3}$

(D) $-\frac{23}{3}$

Solution

Factor: $x^2 - 7x + 10 = (x-2)(x-5)$. On $[0,2]$, both factors satisfy $x - 2 \leq 0$ and $x - 5 < 0$, so their product is ≥ 0 on $[0,2]$ (with equality only at $x = 2$). Hence $|x^2 - 7x + 10| = x^2 - 7x + 10$ on $[0,2]$, and

$$\int_0^2 (x^2 - 7x + 10)\, dx = \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x\right]_0^2 = \frac{8}{3} - 14 + 20 = \frac{8}{3} + 6 = \frac{26}{3}.$$

Final Answer

$$\boxed{\text{Option C: } \tfrac{26}{3}}$$

Common Trap / Note

Verifying the sign of the quadratic on the interval is essential before dropping the absolute value.

Question 5

Paper: PEA **Year:** 2026 **Topic:** Real Analysis
Subtopic/Concept: Rolle's theorem applied to an antiderivative **Difficulty:** Moderate
Status: Verified

Question

Let $f : [0,1] \to \mathbb{R}$ be continuous with $\displaystyle\int_0^1 f(t)\, dt = 1$, and let

$$P(x) = \sum_{k=1}^{n} a_k x^k, \qquad \sum_{k=1}^{n} a_k = 1.$$

Then there exists $c \in (0,1)$ such that

(A) $f(c) = 2\sum_{k=1}^{n} a_k$

(B) $f(c) = \int_0^1 P(t)\, dt$

(C) $f(c) = P'(c)$

(D) $f(c) = P(c)$

Solution

Define

$$\Phi(x) \;=\; \int_0^x f(t)\, dt \;-\; P(x), \qquad x \in [0,1].$$

Then Φ is continuous on $[0,1]$, differentiable on $(0,1)$, and

$$\Phi(0) = 0 - P(0) = 0, \qquad \Phi(1) = \int_0^1 f(t)\, dt - P(1) = 1 - \sum_{k=1}^{n} a_k = 1 - 1 = 0.$$

By Rolle's theorem, there exists $c \in (0,1)$ with $\Phi'(c) = 0$, i.e.

$$f(c) - P'(c) = 0 \quad \Longleftrightarrow \quad f(c) = P'(c).$$

Final Answer

$$\boxed{\text{Option C: } f(c) = P'(c)}$$

Common Trap / Note

Note $P(0) = 0$ because the sum starts at $k = 1$. This is essential for $\Phi(0) = 0$.

Question 6

Paper: PEA **Year:** 2026 **Topic:** Real Analysis
Subtopic/Concept: Cauchy mean value theorem **Difficulty:** Moderate **Status:** Verified

Question

Let $f : [0, \pi/4] \to \mathbb{R}$ be continuous. Then there exists $c \in (0, \pi/4)$ such that

(A) $f(c) = \int_0^{\pi/4} f(t)\,dt$

(B) $f(c) = 2\cos(2c)\int_0^{\pi/4} f(t)\,dt$

(C) $f(c) = \sin(2c)$

(D) $f(c) = \sin(2c)\int_0^{\pi/4} f(t)\,dt$

Solution

Define

$$F(x) = \int_0^x f(t)\,dt, \qquad G(x) = \sin(2x), \qquad x \in [0, \pi/4].$$

Both are continuous on $[0, \pi/4]$ and differentiable on $(0, \pi/4)$ with $F'(x) = f(x)$ and $G'(x) = 2\cos(2x)$. Moreover $G'(x) = 2\cos(2x) \neq 0$ on $(0, \pi/4)$, since $0 < 2x < \pi/2$ there.

By Cauchy's mean value theorem, there exists $c \in (0, \pi/4)$ such that

$$\frac{F(\pi/4) - F(0)}{G(\pi/4) - G(0)} = \frac{F'(c)}{G'(c)},$$

i.e.

$$\frac{\int_0^{\pi/4} f(t)\,dt}{\sin(\pi/2) - 0} = \frac{f(c)}{2\cos(2c)},$$

which simplifies to

$$f(c) = 2\cos(2c)\int_0^{\pi/4} f(t)\,dt.$$

Final Answer

$$\boxed{\text{Option B: } f(c) = 2\cos(2c)\int_0^{\pi/4} f(t)\,dt}$$

Common Trap / Note

The integral on the right is a constant; the c-dependence enters only through $\cos(2c)$.

Question 7

Paper: PEA **Year:** 2026 **Topic:** Real Analysis
Subtopic/Concept: FTC applied to an identity **Difficulty:** Easy **Status:** Verified

Question

Suppose $f : [a, b] \to \mathbb{R}$ is continuous and

$$\int_a^x f(t)\,dt \;=\; \int_x^b f(t)\,dt \quad \text{for all } x \in [a, b].$$

Then $f(x)$ must be

(A) A non-zero constant

(B) A linear function

(C) Identically zero

(D) An odd function about $\dfrac{a+b}{2}$

Solution

Differentiate both sides with respect to x using the fundamental theorem of calculus:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x), \qquad \frac{d}{dx}\int_x^b f(t)\,dt = -f(x).$$

Hence $f(x) = -f(x)$, so $2f(x) = 0$, i.e. $f(x) = 0$ for every $x \in [a, b]$.

Final Answer

$$\boxed{\text{Option C: identically zero}}$$

Common Trap / Note

An odd function about $\frac{a+b}{2}$ would satisfy the identity only at the single point $x = \frac{a+b}{2}$, not for all x.

Question 8

Paper: PEA **Year:** 2026 **Topic:** Calculus
Subtopic/Concept: L'Hôpital's rule, symmetry of odd functions **Difficulty:** Hard **Status:** Verified

Question

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous odd function, vanishing exactly at one point and satisfying $f(1) = \frac{1}{2}$. If

$$\lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14},$$

where

$$F(x) = \int_{-1}^{x} f(t)\,dt, \qquad G(x) = \int_{-1}^{x} t\,|f(f(t))|\,dt,$$

then the value of $f\left(\frac{1}{2}\right)$ is

(A) $\frac{1}{7}$

(B) 7

(C) -7

(D) $\frac{7}{2}$

Solution

Both numerator and denominator vanish at $x = 1$. Since f is odd, $\int_{-1}^{1} f(t)\,dt = 0$, so $F(1) = 0$. For $G(1)$, note that f odd implies $f(f(-t)) = f(-f(t)) = -f(f(t))$, so $|f(f(\cdot))|$ is even. Hence $t\,|f(f(t))|$ is an odd function of t, and

$$G(1) = \int_{-1}^{1} t\,|f(f(t))|\,dt = 0.$$

Thus the limit is of indeterminate form $0/0$, and by L'Hôpital's rule:

$$\frac{1}{14} = \lim_{x\to 1} \frac{F'(x)}{G'(x)} = \lim_{x\to 1} \frac{f(x)}{x\,|f(f(x))|} = \frac{f(1)}{1 \cdot |f(f(1))|} = \frac{1/2}{|f(1/2)|}.$$

Therefore $|f(1/2)| = 14 \cdot \frac{1}{2} = 7$.

Determining the sign. Since f is continuous and vanishes only at one point, and f odd implies $f(0) = 0$, that single zero must be at 0. Thus f has constant sign on $(0, \infty)$. From $f(1) = \frac{1}{2} > 0$, we conclude $f > 0$ on $(0, \infty)$, so $f(1/2) > 0$, giving $f(1/2) = 7$.

Final Answer

$$\boxed{\text{Option B: 7}}$$

Common Trap / Note

The symmetry argument is the key step: without it the denominator $G(1)$ does not obviously vanish and L'Hôpital cannot be applied.

Question 9

Paper: PEA **Year:** 2026 **Topic:** Statistics
Subtopic/Concept: Interpretation of zero correlation **Difficulty:** Easy **Status:** Verified

Question

If the correlation between X and Y is zero, then which of the following is always true?

(A) They are independent

(B) There is no linear relationship

(C) They cannot be functionally related

(D) All of the above

Solution

Zero correlation is by definition the absence of any linear association. Counterexamples eliminate the other options:

- Zero correlation does not imply independence (e.g. $Y = X^2$ with X symmetric about 0 yields zero correlation but X, Y are dependent).
- In the same example, X and Y are functionally related.

Thus only "no linear relationship" is always true.

Final Answer

$$\boxed{\text{Option B: no linear relationship}}$$

Common Trap / Note

"Uncorrelated" is strictly weaker than "independent" except for jointly Gaussian variables.

Question 10

Paper: PEA **Year:** 2026 **Topic:** Regression
Subtopic/Concept: OLS passes through the mean point **Difficulty:** Easy **Status:** Verified

Question

In simple linear regression $Y = a + bX$, the least squares line based on n data points always passes through

(A) $(0, 0)$

(B) $\left(\frac{1}{n}, \frac{1}{n}\right)$

(C) $(1, 1)$

(D) None of the previous

Solution

The OLS line always passes through the point $(\bar{X}, \bar{Y})$, the sample means of X and Y. None of the offered points coincides with $(\bar{X}, \bar{Y})$ in general.

Final Answer

$$\boxed{\text{Option D: None of the previous}}$$

Common Trap / Note

No major trap beyond standard calculation care.

Question 11

Paper: PEA **Year:** 2026 **Topic:** Hypothesis Testing
Subtopic/Concept: Trade-off between α and power **Difficulty:** Easy **Status:** Verified

Question

Given sample size n, reducing the significance level α

(A) Increases Type I error

(B) Decreases Type II error

(C) Increases power

(D) Decreases power

Solution

By definition $\alpha = P(\text{Type I})$, so smaller α means fewer Type I errors. For a fixed n, the rejection region shrinks, increasing the probability of failing to reject the null when it is false, i.e. Type II error β rises and power $1 - \beta$ falls.

Final Answer

$$\boxed{\text{Option D: decreases power}}$$

Common Trap / Note

Power can only be raised (for fixed effect size) by increasing n or α.

Question 12

Paper: PEA **Year:** 2026 **Topic:** Random Variables
Subtopic/Concept: Moments of a Bernoulli variable **Difficulty:** Easy **Status:** Verified

Question

If $X \sim \text{Bernoulli}(p)$, then $E(X^2)$ equals

(A) p^2

(B) p

(C) $p(1-p)$

(D) $p+p^2$

Solution

Since $X \in \{0,1\}$, we have $X^2 = X$, hence $E(X^2) = E(X) = p$.

Final Answer

$$\boxed{\text{Option B: } p}$$

Common Trap / Note

Note $\text{Var}(X) = p(1-p) = E(X^2) - (EX)^2$.

Question 13

Paper: PEA **Year:** 2026 **Topic:** Statistics
Subtopic/Concept: Unbiased estimation for Poisson **Difficulty:** Moderate **Status:** Verified

Question

For $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$ (i.i.d.), which is unbiased for λ^2?

(A) $\bar{X}^2$

(B) $\bar{X}$

(C) $\bar{X}^2 - \dfrac{\bar{X}}{n}$

(D) $\bar{X}^2 - \bar{X}$

Solution

For Poisson, $E(\bar{X}) = \lambda$ and $\text{Var}(\bar{X}) = \lambda/n$. Therefore

$$E(\bar{X}^2) = \text{Var}(\bar{X}) + (E\bar{X})^2 = \frac{\lambda}{n} + \lambda^2.$$

Consequently

$$E\left(\bar{X}^2 - \frac{\bar{X}}{n}\right) = \frac{\lambda}{n} + \lambda^2 - \frac{\lambda}{n} = \lambda^2.$$

Hence $\bar{X}^2 - \bar{X}/n$ is unbiased for λ^2.

Final Answer

$$\boxed{\text{Option D: } \bar{X}^2 - \frac{\bar{X}}{n}}$$

Common Trap / Note

Option (D)-correction subtracts $\bar{X}/n$, *not* $\bar{X}$. The bias of $\bar{X}^2$ is λ/n, not λ.

Question 14

Paper: PEA **Year:** 2026 **Topic:** Descriptive Statistics
Subtopic/Concept: Robustness of median vs. mean **Difficulty:** Easy **Status:** Verified

Question

A dataset consists of 9 observations arranged in increasing order $x_1 \leq x_2 \leq \cdots \leq x_9$ with mean 20 and median 18. If x_9 is replaced by a very large number, which must be true?

(A) Both the mean and the median increase

(B) The mean increases but the median remains unchanged

(C) The median increases but the mean remains unchanged

(D) Both the mean and the median remain unchanged

Solution

The median of 9 ordered observations is x_5, which is unaffected by changing the largest observation x_9 (the ordering is preserved). The mean equals the sum divided by 9, so increasing x_9 raises the sum and hence the mean.

Final Answer

$$\boxed{\text{Option B}}$$

Common Trap / Note

The median is robust to extreme values; the mean is not.

Question 15

Paper: PEA **Year:** 2026 **Topic:** Probability
Subtopic/Concept: Bayes' rule **Difficulty:** Easy **Status:** Verified

Question

A restaurant has 60% vegetarian customers (of whom 30% order dessert) and 40% non-vegetarian (of whom 50% order dessert). Given that a randomly selected customer ordered dessert, what is the probability they ordered vegetarian?

(A) 0.375

(B) 0.474

(C) 0.600

(D) 0.750

Solution

By Bayes' theorem

$$P(V \mid D) = \frac{P(D \mid V)\, P(V)}{P(D \mid V)P(V) + P(D \mid V^c)P(V^c)} = \frac{0.3 \cdot 0.6}{0.3 \cdot 0.6 + 0.5 \cdot 0.4} = \frac{0.18}{0.38} = \frac{9}{19} \approx 0.474.$$

Final Answer

$$\boxed{\text{Option B: 0.474}}$$

Common Trap / Note

No major trap beyond standard calculation care.

Question 16

Paper: PEA **Year:** 2026 **Topic:** Consumer Theory
Subtopic/Concept: Properties of preferences **Difficulty:** Moderate **Status:** Verified

Question

Suppose preferences are represented by $u(x, y) = x - \ln y$ for $x, y > 0$. Then the underlying preference relation must be

(A) Incomplete

(B) Complete but intransitive

(C) Complete and transitive but discontinuous

(D) Complete, transitive and continuous but not convex

Solution

Since u is a real-valued function on $\mathbb{R}^2_{++}$, the induced preference is automatically *complete* and *transitive*. Since u is continuous, the preference is *continuous*.

Convexity. The upper contour set is

$$\{(x, y) : x - \ln y \geq c\} = \{(x, y) : x \geq c + \ln y\}.$$

The boundary $x = c + \ln y$ is *concave* in y (second derivative $-1/y^2 < 0$), so the region lying *above* it is not a convex set. Equivalently, the Hessian

$$\nabla^2 u = \begin{pmatrix} 0 & 0 \\ 0 & 1/y^2 \end{pmatrix}$$

is positive semidefinite, not negative semidefinite, so u is convex (not concave) in y and the preference fails to be convex.

Hence the preference is complete, transitive, continuous, but not convex.

Final Answer

$$\boxed{\text{Option D}}$$

Common Trap / Note

Note that y enters the utility with a *negative* sign through $-\ln y$; so the good y is a "bad" here, and indifference curves slope the wrong way for convexity.

Question 17

Paper: PEA **Year:** 2026 **Topic:** General Equilibrium
Subtopic/Concept: Exchange economy, gains from trade **Difficulty:** Moderate **Status:** Draft

Question

Suppose there are two agents A and B with utilities

$$u_A = x_A + y_A - \tfrac{m}{2}(x_A - y_A)^2, \qquad u_B = x_B + y_B,$$

where x_i and y_i are the amounts of goods X and Y consumed by agent $i \in \{A, B\}$. The total endowments are x and y. Initially A owns the entire endowment of Y and B owns the entire endowment of X. Then A will voluntarily transfer a positive amount of Y to B if

(A) $m > 0$

(B) $m < 0$

(C) $y > 2$

(D) $my > \dfrac{1}{2}$

Solution

Compute A's marginal rate of substitution at the endowment $(x_A, y_A) = (0, y)$:

$$\frac{\partial u_A}{\partial x_A} = 1 - m(x_A - y_A), \qquad \frac{\partial u_A}{\partial y_A} = 1 + m(x_A - y_A).$$

At $(0, y)$:

$$\text{MRS}^A_{x,y} = \frac{\partial u_A/\partial x_A}{\partial u_A/\partial y_A} = \frac{1+my}{1-my}.$$

Agent B's utility is linear with $\text{MRS}^B_{x,y} = 1$.

For A to voluntarily transfer Y to B in exchange for X (so that both agents are made strictly better off), A must value X relative to Y more than B does at the endowment, i.e.

$$\text{MRS}^A_{x,y} > \text{MRS}^B_{x,y} \;=\; 1.$$

Assuming $1 - my > 0$ (so that marginal utilities are positive at the endowment),

$$\frac{1+my}{1-my} > 1 \quad\Longleftrightarrow\quad 1 + my > 1 - my \quad\Longleftrightarrow\quad my > 0.$$

Since the endowment $y > 0$, this reduces to $m > 0$.

Final Answer

$$\boxed{\text{Option A: } m > 0}$$

Common Trap / Note

The condition is on the *sign* of m, not on its magnitude relative to y, because the relevant comparison is between A's MRS at the endowment and B's constant MRS of unity.

Question 18

Paper: PEA **Year:** 2026 **Topic:** Public Goods
Subtopic/Concept: Samuelson condition **Difficulty:** Moderate **Status:** Needs Review

Question

Two agents A and B have utilities

$$u_A = x_A + y, \qquad u_B = 2x_B + y,$$

where x_A, x_B are amounts of a private good and y is the amount of a public good. Production satisfies $y + m(x_A + x_B) = 1$, with $m \in (0, 1)$. Suppose a social planner decides that one unit should be produced. For what value of m will the resulting allocation be Pareto optimal?

(A) $\frac{4}{5}$

(B) $\frac{4}{7}$

(C) $\frac{4}{9}$

(D) $\frac{4}{11}$

Solution

Marginal rates of substitution between the public good y and the private good are

$$\mathrm{MRS}^A_{y,x} = \frac{\partial u_A/\partial y}{\partial u_A/\partial x_A} = 1, \qquad \mathrm{MRS}^B_{y,x} = \frac{\partial u_B/\partial y}{\partial u_B/\partial x_B} = \tfrac{1}{2}.$$

The marginal rate of transformation between y and the private good, from $y + m(x_A + x_B) = 1$, is $\mathrm{MRT}_{y,x} = \frac{1}{m}$ (units of private good required per additional unit of public good).
The Samuelson condition for Pareto optimality with a public good is

$$\mathrm{MRS}^A_{y,x} + \mathrm{MRS}^B_{y,x} \;=\; \mathrm{MRT}_{y,x},$$

which yields

$$1 + \tfrac{1}{2} = \tfrac{1}{m} \quad\Longrightarrow\quad m = \tfrac{2}{3}.$$

This value is not among the options listed. Given the printed answer choices $\{4/5, 4/7, 4/9, 4/11\}$, the question as transcribed is internally inconsistent with the standard Samuelson criterion, so the problem statement (in particular the utility specifications or the production technology) is flagged for review.

Final Answer

$$\boxed{\text{Needs Review (standard Samuelson computation yields } m = \tfrac{2}{3})}$$

Common Trap / Note

The transcribed utilities yield $m = 2/3$, which does not appear among the options. Until the original problem statement (utility functions and/or production technology) is verified, the correct option cannot be uniquely determined.

Question 19

Paper: PEA **Year:** 2026 **Topic:** Market Structure
Subtopic/Concept: Monopoly with discrete buyers **Difficulty:** Moderate **Status:** Verified

Question

Three consumers with valuations v_A, v_B, v_C each wish to buy one unit. The monopolist has zero marginal cost and knows the valuations but must charge a uniform price. It is given that

$$\frac{7v_B}{6} > v_A > v_B > \frac{8v_C}{5} > 0.$$

The profit-maximising price is

(A) v_A

(B) v_B

(C) v_C

(D) Something in (v_C, v_B)

Solution

The monopolist's optimum lies at one of the buyers' valuations (any higher price loses an additional buyer without gain, any lower price wastes surplus). Profit at each:

- Price v_A: only A buys, profit $= v_A$.
- Price v_B: A and B buy, profit $= 2v_B$.
- Price v_C: all three buy, profit $= 3v_C$.

Compare:

$$2v_B \text{ vs } v_A : \quad v_A < \tfrac{7v_B}{6} < 2v_B, \quad \text{so} \quad 2v_B > v_A.$$

$$2v_B \text{ vs } 3v_C : \quad v_B > \tfrac{8v_C}{5}, \quad \text{so} \quad 2v_B > \tfrac{16}{5}v_C > 3v_C.$$

Thus charging v_B yields the highest profit.

Final Answer

$$\boxed{\text{Option B: } p = v_B}$$

Common Trap / Note

The two inequalities $7v_B/6 > v_A$ and $5v_B > 8v_C$ are precisely what is needed for v_B to dominate both alternatives.

Question 20

Paper: PEA **Year:** 2026 **Topic:** Game Theory
Subtopic/Concept: Voluntary contribution to a public good, symmetric Nash equilibrium
Difficulty: Moderate **Status:** Verified

Question

n agents simultaneously choose private consumption x_i and public-good contribution y_i to maximise

$$u_i = x_i \left(\sum_{j=1}^{n} y_j \right),$$

subject to $x_i + y_i = b$, $x_i, y_i \geq 0$. As n increases, the total Nash equilibrium expenditure on the public good

(A) Remains invariant

(B) Monotonically increases and approaches b

(C) Approaches $\dfrac{b}{n}$

(D) Monotonically decreases and approaches 0

Solution

Let $Y_{-i} = \sum_{j \neq i} y_j$. Agent i chooses $y_i \in [0, b]$ to maximise

$$u_i = (b - y_i)(y_i + Y_{-i}).$$

The FOC is

$$-(y_i + Y_{-i}) + (b - y_i) = 0 \quad \Longrightarrow \quad y_i = \frac{b - Y_{-i}}{2}.$$

At a symmetric equilibrium $y_i = y^*$ and $Y_{-i} = (n-1)y^*$:

$$y^* = \frac{b - (n-1)y^*}{2} \quad \Longrightarrow \quad (n+1)y^* = b \quad \Longrightarrow \quad y^* = \frac{b}{n+1}.$$

Therefore total public-good expenditure is

$$Y \;=\; ny^* \;=\; \frac{nb}{n+1}.$$

The map $n \mapsto \dfrac{n}{n+1}$ is strictly increasing and approaches 1 as $n \to \infty$. Hence Y monotonically increases and approaches b.

Final Answer

$$\boxed{\text{Option B}}$$

Common Trap / Note

Per-capita contribution $b/(n+1)$ shrinks, but it shrinks slower than $1/n$, so the aggregate grows.

Question 21

Paper: PEA **Year:** 2026 **Topic:** Public Goods
Subtopic/Concept: Vertical summation of demand, Samuelson condition **Difficulty:** Easy
Status: Verified

Question

A and B are neighbours; the snow plough cannot clear in front of A's house without clearing in front of B's. Inverse demands are $P_A = 12 - q$, $P_B = 8 - q$. Marginal cost is 16. What is the efficient level of provision?

(A) 6

(B) 4

(C) 2

(D) 1

Solution

Since the service is non-rival between A and B, vertical summation gives the aggregate marginal willingness to pay:

$$P_A + P_B \;=\; (12 - q) + (8 - q) \;=\; 20 - 2q.$$

Efficiency requires $\sum P_i = \text{MC}$:

$$20 - 2q \;=\; 16 \quad\Longrightarrow\quad q^* \;=\; 2.$$

Final Answer

$$\boxed{\text{Option C: } q^* = 2}$$

Common Trap / Note

For a public good, demands are summed *vertically* (prices summed at each quantity), not horizontally.

Question 22

Paper: PEA **Year:** 2026 **Topic:** Microeconomics
Subtopic/Concept: Tax incidence **Difficulty:** Easy **Status:** Verified

Question

A lump-sum tax of Rs. 1 is paid by the buyer per unit of a competitive good. The price paid by buyers rises by 80 paise. Then

(A) Demand must be perfectly elastic

(B) Demand must be perfectly inelastic

(C) Demand and supply elasticities must both be positive but finite

(D) Supply must be perfectly elastic

Solution

With elasticities $\varepsilon_D, \varepsilon_S$ (positive numbers), the share of the tax borne by buyers is

$$\frac{\varepsilon_S}{\varepsilon_S + \varepsilon_D}.$$

We are told this share equals 0.8.

- Perfectly elastic demand ($\varepsilon_D = \infty$) gives buyer share 0. Rules out (A).
- Perfectly inelastic demand ($\varepsilon_D = 0$) gives buyer share 1. Rules out (B).
- Perfectly elastic supply ($\varepsilon_S = \infty$) gives buyer share 1. Rules out (D).

A buyer share strictly between 0 and 1 requires both elasticities to be positive and finite.

Final Answer

$$\boxed{\text{Option C}}$$

Common Trap / Note

The incidence formula uses absolute elasticities; the buyer share is decreasing in ε_D and increasing in ε_S.

Question 23

Paper: PEA **Year:** 2026 **Topic:** Consumer Theory
Subtopic/Concept: Engel aggregation **Difficulty:** Moderate **Status:** Verified

Question

A consumer always spends one-fourth of income on X, and the income elasticity of demand for X is 5. Is Y inferior?

(A) Yes

(B) No

(C) Y is inferior iff $p_Y \geq 2p_X$

(D) Y is inferior iff $p_Y \leq p_X/2$

Solution

Let $s_X = 1/4$ and $s_Y = 3/4$ be the budget shares. Engel aggregation states that the share-weighted sum of income elasticities equals 1:

$$s_X\,\eta_X + s_Y\,\eta_Y = 1.$$

Substituting $s_X = 1/4$, $\eta_X = 5$, $s_Y = 3/4$:

$$\tfrac{1}{4}\cdot 5 + \tfrac{3}{4}\,\eta_Y = 1 \quad\Longrightarrow\quad \tfrac{3}{4}\,\eta_Y = -\tfrac{1}{4} \quad\Longrightarrow\quad \eta_Y = -\tfrac{1}{3} < 0.$$

A negative income elasticity means Y is inferior, irrespective of prices.

Final Answer

$$\boxed{\text{Option A: Yes}}$$

Common Trap / Note

Engel aggregation follows from differentiating the budget identity with respect to income.

Question 24

Paper: PEA **Year:** 2026 **Topic:** Consumer Theory
Subtopic/Concept: Aggregate price elasticity as quantity-weighted average **Difficulty:** Easy **Status:** Verified

Question

Half the total quantity is purchased by 75 consumers each with price elasticity 2, and the other half by 25 consumers each with price elasticity 3. What is the aggregate price elasticity for the 100 consumers?

(A) Cannot be determined

(B) 2.25

(C) 2.5

(D) 2.75

Solution

The market elasticity is the quantity-share-weighted average of group elasticities. With each group accounting for 50% of the quantity:

$$\varepsilon = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 3 = 2.5.$$

Final Answer

$$\boxed{\text{Option C: 2.5}}$$

Common Trap / Note

Weights are by *quantity* (or revenue, given a common price), not by *number of consumers*.

Question 25

Paper: PEA **Year:** 2026 **Topic:** International Trade
Subtopic/Concept: Two-country Cobb–Douglas equilibrium **Difficulty:** Moderate **Status:** Verified

Question

Two countries H and F produce single goods with outputs Y_H, Y_F. Take H's good as numeraire and p as the relative price of F's good. Consumers in country i spend a fraction $1/4$ of their expenditure on the foreign good. E_i denotes total expenditure of country i measured in its own good. Given $Y_H = 120$, $Y_F = 100$, $E_H = 80$ and market clearing with $Y_H + pY_F = E_H + pE_F$, find p.

(A) 8

(B) 6

(C) 4

(D) 2

Solution

By the Cobb–Douglas expenditure shares, each country spends 3/4 on its own good and 1/4 on the foreign good.

Market clearing for H's good (measured in units of H's good). Demand by H-consumers: $\frac{3}{4}E_H = \frac{3}{4}\cdot 80 = 60$. Demand by F-consumers: a fraction 1/4 of their total expenditure $p\,E_F$ expressed in H-units, i.e. $\frac{1}{4}\,p\,E_F$. Total demand equals supply Y_H:

$$60 + \tfrac{1}{4}\,p\,E_F \;=\; 120 \quad\Longrightarrow\quad p\,E_F = 240.$$

Budget identity. Total income = total expenditure (in the same units):

$$Y_H + p\,Y_F \;=\; E_H + p\,E_F \quad\Longrightarrow\quad 120 + 100p \;=\; 80 + 240 \;=\; 320.$$

Thus $100p = 200$, hence $p = 2$.

Final Answer

$$\boxed{\text{Option D: } p = 2}$$

Common Trap / Note

Keeping the units straight (own good vs. numeraire) is the only delicate step.

Question 26

Paper: PEA **Year:** 2026 **Topic:** Macroeconomics
Subtopic/Concept: Classical monetary equilibrium with full employment **Difficulty:** Moderate **Status:** Verified

Question

An economy has one good produced one-for-one from labour. The representative consumer has $U\big(C, \frac{M}{P}\big) = \frac{3}{4}\ln C + \frac{1}{4}\ln\frac{M}{P}$, labour endowment 50, money endowment $\bar{M} = 100$, with constant money supply $M = \bar{M}$. If the price is flexible (so labour is fully employed), the equilibrium price is

(A) 2

(B) 4

(C) 6

(D) 8

Solution

With linear technology and perfect competition $P = W$. The consumer's real wealth (measured in units of the good) is

$$\Omega = L + \frac{\bar{M}}{P},$$

where L is labour supplied. Cobb–Douglas preferences imply

$$C = \tfrac{3}{4}\,\Omega, \qquad \tfrac{M}{P} = \tfrac{1}{4}\,\Omega.$$

Full employment means $L = 50$ and goods-market clearing $C = Y = L = 50$:

$$50 = \tfrac{3}{4}\big(50 + \tfrac{100}{P}\big) \iff \tfrac{200}{3} - 50 = \tfrac{100}{P} \cdot \tfrac{3}{4} \cdot \tfrac{4}{3}.$$

Equivalently, the money-market condition

$$\frac{100}{P} = \tfrac{1}{4}\big(50 + \tfrac{100}{P}\big)$$

gives

$$\frac{400}{P} - \frac{100}{P} = 50 \implies \frac{300}{P} = 50 \implies P = 6.$$

Final Answer

$$\boxed{\text{Option C: } P = 6}$$

Common Trap / Note

Either the goods market or the money market suffices (Walras' law guarantees the other clears).

Question 27

Paper: PEA **Year:** 2026 **Topic:** Macroeconomics
Subtopic/Concept: Fixed-price (Keynesian) equilibrium **Difficulty:** Moderate **Status:** Verified

Question

With the same setup as Question 26 but with price fixed at $P = 10$, the equilibrium output is

(A) 15

(B) 20

(C) 25

(D) 30

Solution

With $P = 10$, real money endowment is $\bar{M}/P = 10$. The money-demand condition

$$\frac{M}{P} = \tfrac{1}{4}\Big(L + \tfrac{\bar{M}}{P}\Big)$$

becomes

$$10 = \tfrac{1}{4}(L + 10) \implies L + 10 = 40 \implies L = 30.$$

Since the price is above the market-clearing level $P^* = 6$ (Question 26), the labour market is in excess supply and employment is demand-determined. Output $Y = L = 30$.

Final Answer

$$\boxed{\text{Option D: } Y = 30}$$

Common Trap / Note

At a fixed price above P^*, output is pinned down by the money-market (or equivalently, goods-market) condition, not by the labour supply.

Question 28

Paper: PEA **Year:** 2026 **Topic:** Growth Theory
Subtopic/Concept: Solow model, growth rate of per-capita output **Difficulty:** Moderate
Status: Verified

Question

Solow economy with $Y = K^{1/2}L^{1/2}$, no depreciation, $n = 0.02$, savings rate $s > 0$. If the steady-state capital–labour ratio is $k^* = 9$ and the current ratio is $k = 1$, then the growth rate of $y = Y/L$ at the current date is

(A) 0.02

(B) 0.04

(C) 0.03

(D) Indeterminable

Solution

Per-worker output is $y = k^{1/2}$. The Solow law of motion (with no depreciation) is

$$\dot{k} = s\,k^{1/2} - n\,k.$$

Steady state: $s\,(k^*)^{1/2} = n\,k^* \Rightarrow s = n\,(k^*)^{1/2} = 0.02 \cdot 3 = 0.06$.
Current growth rate of k:

$$\frac{\dot{k}}{k} = \frac{s}{\sqrt{k}} - n = \frac{0.06}{1} - 0.02 = 0.04.$$

Since $y = k^{1/2}$,

$$\frac{\dot{y}}{y} = \tfrac{1}{2} \cdot \frac{\dot{k}}{k} = \tfrac{1}{2} \cdot 0.04 = 0.02.$$

Final Answer

$$\boxed{\text{Option A: } 0.02}$$

Common Trap / Note

Don't forget the factor 1/2 when converting from the growth rate of k to that of $y = k^{1/2}$.

Question 29

Paper: PEA **Year:** 2026 **Topic:** Growth Theory
Subtopic/Concept: Comparative dynamics across Solow economies **Difficulty:** Moderate
Status: Verified

Question

Two Solow economies A, B share the same technology and $n = 0.02$. They have $k_A = 3, k_A^* = 9$ and $k_B = 4, k_B^* = 16$. Then per capita output is growing

(A) Faster in A than in B

(B) Faster in B than in A

(C) At the same rate in A and B

(D) At an indeterminable relative rate

Solution

As in Question 28, $s_i = n\,(k_i^*)^{1/2}$. Thus $s_A = 0.02 \cdot 3 = 0.06$ and $s_B = 0.02 \cdot 4 = 0.08$. The current growth rate of capital per worker is $\dot{k}_i/k_i = s_i/\sqrt{k_i} - n$, so

$$\frac{\dot{k}_A}{k_A} = \frac{0.06}{\sqrt{3}} - 0.02 \approx 0.0346 - 0.02 = 0.0146, \qquad \frac{\dot{k}_B}{k_B} = \frac{0.08}{2} - 0.02 = 0.04 - 0.02 = 0.02.$$

Halving these gives the growth rates of y:

$$\frac{\dot{y}_A}{y_A} \approx 0.0073, \qquad \frac{\dot{y}_B}{y_B} = 0.01.$$

Hence y grows faster in B than in A.

Final Answer

$$\boxed{\text{Option B}}$$

Common Trap / Note

Both economies are below their respective steady states, but B has a higher savings rate and a closer relative gap, so it grows faster.

Question 30

Paper: PEA **Year:** 2026 **Topic:** Intertemporal Choice
Subtopic/Concept: Log utility and the saving function **Difficulty:** Moderate **Status:** Verified

Question

A two-period consumer maximises

$$U(C_1, C_2) = \log C_1 + \tfrac{1}{1+\rho} \log C_2, \qquad \rho > 0,$$

subject to $C_1 + S = w$ and $C_2 = (1 + r)S$, with $w, r > 0$. If w rises by 1% and r falls by 1%, then S

(A) Increases by more than 1%

(B) Increases by 1%

(C) Decreases by 1%

(D) Decreases by more than 1%

Solution

Substitute $C_2 = (1 + r)(w - C_1)$ into U and differentiate w.r.t. C_1:

$$\frac{1}{C_1} - \frac{1}{1+\rho} \cdot \frac{1+r}{C_2} \cdot (1+r) \cdot \frac{1}{1+r} = \frac{1}{C_1} - \frac{1}{(1+\rho)C_2}(1+r) \cdot \frac{1}{1+r} \cdot (1+r).$$

A cleaner derivation: the Euler equation $\frac{C_2}{C_1} = \frac{1+r}{1+\rho}$, combined with the lifetime budget constraint $C_1 + \frac{C_2}{1+r} = w$, gives

$$C_1 + \frac{1}{1+r} \cdot \frac{1+r}{1+\rho} C_1 \;=\; w \quad\Longrightarrow\quad C_1\left(1 + \frac{1}{1+\rho}\right) = w \quad\Longrightarrow\quad C_1 = \frac{(1+\rho)\,w}{2+\rho}.$$

Therefore

$$S = w - C_1 = w\left(1 - \frac{1+\rho}{2+\rho}\right) = \frac{w}{2+\rho}.$$

This is the well-known log-utility result: *saving is proportional to wage and independent of* r (income and substitution effects of an interest-rate change cancel exactly under log preferences).

Since S depends only on w (not on r):

- A 1% rise in w raises S by exactly 1%.
- A 1% fall in r leaves S unchanged.

Therefore S rises by exactly 1%.

Final Answer

Option B: increases by 1%

Common Trap / Note

Under log utility the income and substitution effects of a change in r on saving cancel out exactly; this is a special property of log preferences.

Review Flags

Question 18. As transcribed (with $u_A = x_A + y$, $u_B = 2x_B + y$ and production constraint $y + m(x_A + x_B) = 1$), the Samuelson condition yields $m = 2/3$, which is not among the printed options $\{4/5, 4/7, 4/9, 4/11\}$. The problem statement (likely the utility functions or the production technology in the underlying source) should be verified before finalising the option choice.

All other questions are flagged as either **Verified** or **Draft** (see the Answer-Key Summary at the front of this booklet).

End of PEA Solutions — ISI MSQE 2026, prepared for Statstrive.